

## Topology IB: Exam 9.5.2023

Length of exam: 2h 30min. Aiding material (including calculators) is NOT allowed in the exam.

### Problem 1

- (a) State the definition of a homeomorphism. (2p.)
- (b) Find a homeomorphism  $f: \mathbb{R}^2 \rightarrow H$ , where

$$H := \{(x, y) \in \mathbb{R}^2 : x > 0\}$$

is the right-hand-side half plane. Use the standard Euclidean metric in both  $\mathbb{R}^2$  and  $H$ . (4p.)

### Problem 2

- (a) State the definition of a complete metric space. (2p.)
- (b) Let  $(X, d)$  and  $(Y, d')$  be nonempty metric spaces and  $f: X \rightarrow Y$  be a bijective bilipschitz map. Suppose that  $X$  is complete. Show that also  $Y$  is complete. (4p.)

### Problem 3

- (a) State the definition of a compact metric space. (2p.)
- (b) Is the space

$$X = \{(x, y) \in \mathbb{R}^2 : x^2 + 3y^2 < 1\}$$

compact, when  $X$  is equipped with the standard Euclidean metric? Please explain your argument carefully. (4p.)

### Problem 4

- (a) State the definition of a connected metric space. (2p.)
- (b) Consider the metric space  $(\mathbb{R}, d)$ , where  $d$  is the discrete metric

$$d(x, y) = \begin{cases} 0 & \text{if } x = y; \\ 1 & \text{if } x \neq y. \end{cases}$$

Is  $(\mathbb{R}, d)$  connected? Please explain your argument carefully. (4p.)