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UNIVERSITY OF HELSINKI

Dept of Mathematics and Statistics

MAT12003/AYMAT12003
Probability I
Course exam 2 h 30 min
6.3.2020 at 12:00–14:30

It is allowed to use a calculator in the exam but not any other aiding tools (for example, use of any tables is forbidden).

1. There are several black and white balls to organise a random sample. Three black balls and one white ball are placed in a bowl. One ball is selected at random from the bowl and it is replaced by a ball of the other color (black by white or white by black) which is returned to the bowl. Then a second ball is selected at random from the bowl.
   (a) What is the probability that the second ball is black?
   (b) The second ball is black. What is then the probability that the first ball was black?

2. A city tour organiser has a bus where are 48 seats for travellers. The organiser sells 50 tickets for a tour. On average one customer from 10 customers who have bought a ticket does not show up for the tour. What is the probability that everyone who shows up for the tour will have a seat in the bus?

3. A fair die is tossed 300 times. By using the normal approximation find the probability that outcomes five and six together occur at most 80 times.

4. Let $X \sim \text{Uni}(-1, 1)$.
   (a) What is the probability density function of the random variable $Y = |X + 1|$?
   (b) Find $P(-1 < Y < 1)$.

A table of the values of the cumulative distribution function of the standard normal distribution and a list of frequency and density functions, expectations and variances of some distributions are given in the reverse side of this paper. ↻

Values of the cumulative distribution function Φ of the standard normal distribution; $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}t^2} dt$

x	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,500000	0,503989	0,507978	0,511966	0,515953	0,519938	0,523922	0,527903	0,531881	0,535856
0,1	0,539828	0,543795	0,547758	0,551717	0,555670	0,559618	0,563560	0,567495	0,571424	0,575345
0,2	0,579260	0,583166	0,587064	0,590954	0,594835	0,598706	0,602568	0,606420	0,610261	0,614092
0,3	0,617911	0,621720	0,625616	0,629300	0,633072	0,636831	0,640576	0,644309	0,648027	0,651732
0,4	0,655422	0,659097	0,662757	0,666402	0,670031	0,673645	0,677242	0,680822	0,684386	0,687933
0,5	0,691462	0,694974	0,698468	0,702944	0,705402	0,708840	0,712260	0,715661	0,719043	0,722405
0,6	0,725747	0,729069	0,732371	0,735653	0,738914	0,742154	0,745373	0,748571	0,751748	0,754903
0,7	0,758036	0,761148	0,764238	0,767305	0,770350	0,773373	0,776373	0,779350	0,782305	0,785236
0,8	0,788145	0,791030	0,793892	0,796731	0,799546	0,802338	0,805106	0,807850	0,810570	0,813267
0,9	0,815940	0,818589	0,821214	0,823814	0,826391	0,828944	0,831472	0,833977	0,836457	0,838913
1,0	0,841345	0,843752	0,846136	0,848495	0,850830	0,853141	0,855428	0,857690	0,859929	0,862143
1,1	0,864334	0,866500	0,868643	0,870762	0,872857	0,874928	0,876976	0,879000	0,881000	0,882977
1,2	0,884930	0,886861	0,888768	0,890651	0,892512	0,894350	0,896165	0,897958	0,899727	0,901475
1,3	0,903200	0,904902	0,906582	0,908241	0,909877	0,911492	0,913085	0,914656	0,916207	0,917736
1,4	0,919243	0,920730	0,922196	0,923642	0,925066	0,926471	0,927855	0,929219	0,930563	0,931889
1,5	0,933193	0,934478	0,935744	0,936992	0,938220	0,939429	0,940620	0,941792	0,942947	0,944083
1,6	0,945201	0,946301	0,947384	0,948449	0,949497	0,950528	0,951543	0,952540	0,953521	0,954486
1,7	0,955434	0,956367	0,957284	0,958185	0,959070	0,959941	0,960796	0,961636	0,962462	0,963273
1,8	0,964070	0,964852	0,965620	0,966375	0,967116	0,967843	0,968557	0,969258	0,969946	0,970621
1,9	0,971283	0,971933	0,972571	0,973197	0,973810	0,974412	0,975002	0,975581	0,976148	0,976704
2,0	0,977250	0,977784	0,978308	0,978822	0,979325	0,979818	0,980301	0,980774	0,981237	0,981691
2,1	0,982136	0,982571	0,982997	0,983414	0,983823	0,984222	0,984614	0,984997	0,985371	0,985738
2,2	0,986097	0,986447	0,986791	0,987126	0,987454	0,987776	0,988089	0,988396	0,988696	0,988989
2,3	0,989276	0,989556	0,989830	0,990097	0,990358	0,990613	0,990862	0,991106	0,991344	0,991576
2,4	0,991802	0,992024	0,992240	0,992451	0,992656	0,992857	0,993053	0,993244	0,993431	0,993613
2,5	0,993790	0,993963	0,994132	0,994297	0,994457	0,994614	0,994766	0,994915	0,995060	0,995201
2,6	0,995339	0,995473	0,995604	0,995731	0,995855	0,995975	0,996093	0,996207	0,996319	0,996427
2,7	0,996533	0,996636	0,996736	0,996833	0,996928	0,997020	0,997110	0,997197	0,997282	0,997365
2,8	0,997445	0,997523	0,997599	0,997673	0,997744	0,997814	0,997882	0,997948	0,998012	0,998074
2,9	0,998134	0,998193	0,998250	0,998305	0,998359	0,998411	0,998462	0,998511	0,998559	0,998605
	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
3,0	0,998650	0,999032	0,999313	0,999517	0,999663	0,999767	0,999841	0,999892	0,999928	0,999952

Frequency and density functions, expectations and variances of distributions

$$X \sim \text{Bernoulli}(p) \implies P(X = k) = p^k(1-p)^{1-k},\ k = 0, 1;$$
$$EX = p \text{ and } D^2X = p(1-p).$$

$$X \sim \text{Bin}(n, p) \implies P(X = k) = \binom{n}{k} p^k(1-p)^{n-k},\ k = 0, 1, \ldots, n;$$
$$EX = np \text{ and } D^2X = np(1-p).$$

$$X \sim \text{Hyperg}(N, K, n) \implies P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}},\ k = 0, 1, \ldots, n;$$
$$EX = n\frac{K}{N} \text{ and } D^2X = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}.$$

$$X \sim \text{Geom}(p) \implies P(X = k) = p(1-p)^k,\ k = 0, 1, 2, \ldots;$$
$$EX = \frac{1-p}{p} \text{ and } D^2X = \frac{1-p}{p^2}.$$

$$X \sim \text{Poisson}(\lambda) \implies P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!},\ k = 0, 1, 2, \ldots;\ EX = \lambda \text{ and } D^2X = \lambda.$$

$$X \sim \text{Uni}(a, b) \implies f(x) = \begin{cases} \frac{1}{b-a}, & x \in (a, b), \\ 0, & \text{otherwise;} \end{cases} \quad EX = \frac{a+b}{2} \text{ and } D^2X = \frac{(b-a)^2}{12}.$$

$$X \sim \text{Exp}(\lambda) \implies f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise;} \end{cases} \quad EX = \frac{1}{\lambda} \text{ and } D^2X = \frac{1}{\lambda^2}.$$

$$X \sim N(\mu, \sigma^2) \implies f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},\ x \in \mathbb{R};\ EX = \mu \text{ and } D^2X = \sigma^2.$$