DEPARTMENT OF MATHEMATICS AND STATISTICS
TIME SERIES ANALYSIS I -COURSE 2024
Exam 23rd of January 2025

1. a) *(2 points)* Explain what general properties we have to assume for the underlying stochastic process to fulfil in order to use it as basis of a statistical model? Define such properties carefully.

   b) *(4 points)* Give an example of a class of stochastic processes that have a time varying conditional mean (expectation). Explain what restrictions need to be placed on the model parameters in that case in order the model to be statistically feasible and also explain the reasoning behind every restriction needed.

2. *(6 points)* Define autocorrelation function and partial autocorrelation function and explain how they can be employed to find adequate numbers of lags in the AR(p), MA(q) and ARMA(p,q) models.

3. (a) *(1 point)* Define the forecasting principle that minimizes the MSPE (mean square prediction error).

   (b) *(2 points)* Derive h-step forecasts for observations y_{t+h}, $h = 1, 2, ..., H$, when the forecasts are based on variables y_{t-j}, $j \geq 0$ and ARMA(p,q) is the data generating process.

   (c) *(3 points)* Derive the prediction intervals for the forecasts in the ARMA(p,q) process case, when the innovations are i.i.d. with normal distribution.

4. (a) *(2 points)* Derive the conditional log-likelihood function of a time series model.

   (b) *(2 points)* Derive the conditional log-likelihood function and the conditional MLE in the AR(p) model, when the innovations are i.i.d. with normal distribution.

   (c) *(2 points)* Derive the conditional log-likelihood function and the conditional MLE in the GARCH(1,1) model.

Recall that the density of a variable $y_t \sim N(\mu, \sigma^2)$ is

$$f_{y_t}(y_t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big\{ -\frac{1}{2\sigma^2}(y_t - \mu)^2 \Big\}.$$