Risk theory 10.1.2018
General examination 3h 30min
Basic calculator allowed

1. Define the mixed Poisson variable and determine its moment generating function in terms of the generating functions of its building blocks (assuming that the relevant generating functions are finite).

2. Let Z describe the original claim size of the company and let $Z(M)$ be the share of the insurer after an XL-contract with the limit $M > 0$. Assume that the variance $\sigma^2 = \operatorname{Var} Z$ is finite. Let $\sigma^2(M)$ be the variance of $Z(M)$.

a) Prove that $\sigma^2(M) \leq \sigma^2$.

b) Prove that $\sigma^2(M)$ is increasing in M.

3. Claims of a company occur according to a Poisson process with the constant intensity λ and the claim sizes all equal 1. The reporting delays are i.i.d. random variables and they are independent of the occurrence process. The compensations are paid after a years from the reporting times where $a \in (0, 1)$ is a constant. The distribution function of the reporting delay is G. The company starts the insurance business at time 0. Let $t \geq 1$.

a) Determine the expectation of the outstanding claims at time t (the consideration is made at time 0).

b) At time t, the company observes that m claims have been paid and that $m + n$ have been reported where $m, n \in \mathbb{N} \cup \{0\}$. Based on this information, determine the conditional expectation of the outstanding claims at time t.

4. The yearly net payouts of the company are i.i.d $N(\mu, \sigma)$-distributed random variables where $\mu \in (-\infty, 0)$ and $\sigma \in (0, \infty)$. The initial capital of the company is $U_0 > 0$. Let T be the time of ruin and let $x \in (0, -1/\mu)$. Prove that

$$\mathbb{P}(T \leq xU_0) \leq e^{-(1-\mu x)^2 U_0/(2x\sigma^2)}.$$

Hint: the cumulant generating function c of the $N(\mu, \sigma)$-distribution is determined by $c(t) = \mu t + \sigma^2 t^2/2$ for $t \in \mathbb{R}$.

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1. Lecture notes, section 3.2.

2. a) Problem solving 7, exam 5

   b) If $M' < M$ then $Z(M') = \min(M', Z(M))$ so that by a) $\sigma^2(M') \le \sigma^2(M)$.

3. a) The outstanding claims are those which will be reported during $(t-a, \infty)$. The expectation is

$$\int_0^t \lambda(1 - G(t-a-s))\,ds$$

   b) The outstanding claims are those which have been reported during $(t-a, t]$ (there are n such claims) plus the claims which will be reported after t, the expectation of them is

$$\int_0^t \lambda(1 - G(t-s))\,ds.$$

   The sum is also the conditional expectation because the reporting process is Poisson (the increments are independent).

4. By the notations of the lecture notes, $\bar{s} = \infty$,

$$\underline{x} = \lim_{s\to\infty} c'(s)^{\frac{1}{}} = 0, \quad R = -\frac{2\mu}{\sigma^2}, \quad \mu_T = -\frac{1}{\mu}.$$

Theorem 9.1.2 applies for $x \in (0, -\frac{1}{\mu})$ with

$$c^*\left(\frac{1}{x}\right) = \frac{t_x}{x} - c(t_x), \qquad t_x = \frac{1-\mu x}{x\sigma^2},$$

$$c^*\left(\frac{1}{x}\right) = \frac{(1-\mu x)^2}{2\sigma^2 x^2},$$

$$\mathbb{P}(T \le x U_0) \le e^{-x c^*(\frac{1}{x}) U_0} = e^{-\frac{(1-\mu x)^2 U_0}{2x\sigma^2}}.$$