Risk theory 13.12.2017
General examination 3h 30min
Basic calculator allowed

1. Define the compound variable and derive its expectation in terms of the moments of its building blocks (assuming that appropriate expectations are finite).

2. The total claim amount X of a company has a compound Poisson distribution. The yearly premium is $P = (1 + v)\mathbb{E}(X)$ and the initial capital is U_0. The Poisson parameter is λ and the claims sizes are exponentially distributed with the parameter μ. The company is regarded as solvent if the one year ruin probability is at most 0.01. Prove that the company is not solvent.

The company changes all the insurance contracts such that the company will only pay the parts of the claims which exceed the limit M (if the claim size is Z then the compensation is $(Z - M)\mathbb{1}(Z - M > 0)$). Let X_M be the resulting total claim amount. The new premium will be $P_M = (1 + v)\mathbb{E}(X_M)$. Prove that the company is solvent after the change.

The values of the parameters are $U_0 = 20$, $\lambda = 200$, $\mu = 1$, $v = 0.1$ and $M = 2$. The normal approximation can be used to approximate compound Poisson distributions ($\phi(2.33) = 0.99$ where ϕ is the standard normal distribution function).

3. Claims of a company occur according to a Poisson process with the constant intensity λ and the claim sizes all equal 1. The reporting delays are i.i.d. random variables and they are independent of the occurrence process. The compensations are paid after a years from the reporting times where $a \in (0, 1)$ is a constant. The distribution function of the reporting delay is G. The company starts the insurance business at time 0.

a) Determine the expectation of the outstanding claims at time 1 (the consideration is made at time 0).

b) At time 1, the company observes that m claims have been paid and that $m + n$ have been reported where $m, n \in \mathbb{N} \cup \{0\}$. Based on this information, determine the conditional expectation of the outstanding claims at time 1.

4. The yearly net payouts $\xi_1, \xi_2, \ldots$ of the company are i.i.d. random variables and the initial capital is U_0 Assume that $\xi_i = X_i - P$ where P is a constant and X_i has the compound mixed Poisson distribution with the parameter (λ, Q, S). Assume that $P > \lambda a_1$ where $a_1 \in (0, \infty)$ is the mean of the claim size. We consider λ, S and P as fixed. For a given mixing variable Q, denote by $R(Q)$ the Lundberg exponent associated with the net payout process. Prove that $R(Q) \leq R$ where R is the Lundberg exponent in the case where $\mathbb{P}(Q = 1) = 1$. The moment generating functions of the claim size distribution and the mixing variable are assumed to be finite everywhere.

1. Lecture notes, Chapter 4.

$$\mathbb{E}(Z) = \mathbb{E}(\mathbb{E}(Z \mid K)) = \mathbb{E}(K\,\mathbb{E}(Z)) = \mathbb{E}(K)\,\mathbb{E}(Z)$$

2. $a_1 = \frac{1}{\mu}$, $a_2 = \frac{1}{\mu^2}$, $\mathbb{E}(Z) = \frac{\lambda}{\mu}$, $\operatorname{Var} Z = \frac{2\lambda}{\mu^2}$,

$$\mathbb{P}(Z > U_0 + P) \approx 1 - \Phi\left(\frac{U_0 + \frac{\lambda v}{\mu}}{\sqrt{2\lambda/\mu^2}}\right) = 1 - \Phi(2) > 1 - \Phi(2.33) = 0.01.$$

After the change, Z_M is compound Poisson $-(\lambda, S_0)$ where S_0 is the d.f. of $(Z-M)\,1(Z-M)$, or compound Poisson $-(\lambda e^{-\mu M}, S)$ by Ex 2.1.1, S d.f. of Z. Thus with $\lambda_M = \lambda e^{-\mu M}$,

$$\mathbb{P}(Z_M > U_0 + P_M) \approx 1 - \Phi\left(\frac{U_0 + \lambda_M v/\mu}{\sqrt{2\lambda_M/\mu^2}}\right) \approx 1 - \Phi(3.09) < 0.01.$$

3. a) The outstanding claims are those which will be reported during $(1-a, \infty)$. The expectation is

$$\int_0^1 \lambda\,(1 - G(1-a-s))\,ds$$

b) The outstanding claims are those which have been reported during $(1-a, 1]$ + the claims to be reported after 1. Write $U = \{\text{outstanding claims at } 1\}$. We have to calculate

$$\mathbb{E} = \mathbb{E}(U \mid V_1(1-a) = m,\ V_1(1) - V_1(1-a) = n)$$

By the above discussion,

$$\begin{aligned} U &= V_1(1) - V_1(1-a) + V_1(\infty) - V_1(1) \\ &= V_1(\infty) - V_1(1-a) \qquad (\text{as in part a})) \end{aligned}$$

$$\Rightarrow \mathbb{E} = n + \mathbb{E}(V_1(\infty) - V_1(1) \mid V_1(1) = m+n)$$

$$= n + \int_0^1 \lambda\,(1 - G(1-s))\,ds,$$

4. $\mathbb{E}(e^{s\xi}) = \mathbb{E}(e^{\lambda(M_Z(s)-1)Q})\, e^{-Ps}$

$$\overset{\text{Jensen}}{\geq} e^{\lambda(M_Z(s)-1)} \cdot e^{-Ps}$$

The r.h.s. is the mgf in the case where $Q \equiv 1$.

Thus $C(s) \geq C_0(s)$, where $C_0(s) = \lambda(M_Z(s)-1) - Ps$.
The Lundberg exponents exist and necessarily $R(Q) \leq R$.
See also exam 2 at Problem solving 11.