Risk theory 7.2.2018
General examination 3h 30min
Basic calculator allowed

1. Assume that the total claim amount X_i has a compound mixed Poisson distribution, $i = 1, 2$. The associated claim size distribution function is S for both of the variables. Assume that X_1 and X_2 are independent. Prove that $X_1 + X_2$ has a compound mixed Poisson distribution.

2. The capital requirement of every insurance company in the market is determined by the normal approximation of the total claim amount of the forthcoming year. To be solvent, the initial capital together with the premium of the forthcoming year must suffice for the compensations with the probability $\geq 1 - \epsilon$. Assume that two companies in the market are both solvent. Prove that if the companies are merged then also the resulting company is solvent. Relevant moments of the total claim amounts are assumed to be finite.

3. The occurrence process of the claims is a Poisson process with the constant intensity $\lambda > 0$. The reporting delays are i.i.d. random variables and they are independent of the occurrence process. The distribution of the reporting delays is exponential with the parameter μ. The claim sizes all equal 1 and the compensations are paid at the reporting times.

The company starts the insurance business at time 0. At time 1, the company stops the business but has to pay the claims which have occurred in the time interval $(0, 1)$. No other money flows take place after time 1. The capital of the company at time 1 is U. To be solvent, the capital must suffice for the outstanding claims with the probability $\geq 1 - \epsilon$.

a) Is the company solvent at time 1?

b) At time 2, the company observes that N claims have been paid during the time interval $[1, 2)$. Based on this information, determine the conditional probability that the company is able to pay its outstanding claims. Is the company solvent at time 2?

Normal approximation can be used to Poisson distributions. The values of the parameters are $\lambda = 100$, $\mu = 1$, $U = 60$ and $N = 40$. Furthermore, $\epsilon = 0.01$ and $\phi(2.33) = 0.99$ where ϕ is the standard normal distribution function.

4. The yearly total claim amounts $X_1, X_2, \ldots$ are i.i.d. and have the compound Poisson distribution such that the Poisson parameter is λ and the claim sizes are exponentially distributed with the parameter μ. The yearly premium is $(1 + v)\lambda/\mu$. Let T be the time of ruin. Prove that $\mathbb{P}(T < \infty) \leq 0.01$ for every $\lambda > 0$. The other parameters are $\mu = 1$ and $v = 0.05$, and the initial capital of the company is 100.

Risk theory, 7.2.2018

1. $\mathbb{E}(e^{s(Z_1+Z_2)}) = M_{Q_1}(\lambda_1(M_Z(s)-1)) M_{Q_2}(\lambda_2(M_Z(s)-1))$

where Z_i has the compound mixed Poisson distribution with the parameter (λ_i, Q_i, S), and M_Z is the m.g.f. associated with S. We can replace Q_i by Q_i' where Q_i' has the same distribution as Q_i, and $Q_1' \perp\!\!\!\perp Q_2'$. Thus

$$\mathbb{E}(e^{s(Z_1+Z_2)}) = \mathbb{E}(e^{\lambda_1(M_Z(s)-1)Q_1'})\,\mathbb{E}(e^{\lambda_2(M_Z(s)-1)Q_2'})$$

$$= \mathbb{E}\left(e^{\frac{\lambda_1 Q_1' + \lambda_2 Q_2'}{\lambda_1+\lambda_2}(\lambda_1+\lambda_2)(M_Z(s)-1)}\right)$$

which is the m.g.f. of a compound mixed Poisson variable with the parameter

$$\left(\lambda_1+\lambda_2, \frac{\lambda_1 Q_1' + \lambda_2 Q_2'}{\lambda_1+\lambda_2}, S\right)$$

2. Lecture notes, section 6.2.3

3. a) The outstanding claims at time 1 has the Poisson distribution with the parameter

$$\lambda \int_0^1 (1-G(1-s))ds, \qquad G(z) = 1-e^{-\mu z}, \ \lambda = 100, \ \mu = 1,$$

$$= 100(1-e^{-1}) \approx 63.2 > U = 60,$$

the company is not solvent, the ruin probability is $\geq \frac{1}{2}$.

b) The outstanding claims at time 2 has the Poisson distribution with the parameter

$$\lambda \int_0^1 (1-G(2-s))ds \approx 23.3$$

The available capital is $U - 40 = 20$, the company is not solvent;

4. $\xi = X_1 - \frac{(1+v)\lambda}{\mu}$,

$$\mathbb{E}(e^{s\xi}) = e^{\lambda(M_{Z}(s)-1)} \, e^{-\frac{(1+v)\lambda}{\mu} s}$$

$$= e^{\lambda\left(\frac{s}{\mu - s} - \frac{(1+v)s}{\mu}\right)} = 1$$

if $s = 0$ or $s = \frac{\mu v}{1+v} \doteq R$, (Lundberg exponent)

$\mathbb{P}(\tau < \infty) \leq e^{-R u_0} \approx 0{,}008$, when $v = 0{,}05$, $\mu = 1$, $u_0 = 109$