Risk Theory, 27.3.2024
General Examination
Basic calculator allowed

Problem 1. Suppose X is a total claim amount which has a compound mixed Poisson distribution with parameter (λ, Q, S) and the moment generating functions of Q and the claim size are finite in a neighborhood of the origin. Calculate the expectation and variance of X using λ and suitable moments of Q and the claim sizes.

Problem 2. Are the following Statements i)-iii) **TRUE** or **FALSE**? Give a short explanation for your reasoning. In Statements i)-ii), the process $\{K(t) \mid t \geq 0\} = \{K(t)\}$ is a homogeneous Poisson process with intensity $\lambda > 0$.

i) The probability that $\{K(t)\}$ has no jumps in $(0,2]$ is $e^{-\lambda}$.

ii) The probability that $\{K(t)\}$ has exactly one jump in $(0,1]$ and $(1,2]$ is strictly smaller than the probability of $\{K(t)\}$ having exactly two jumps in $(0,2]$.

iii) Suppose $X \geq 0$ is a random variable. If $\mathbb{E}(e^{sX}) < \infty$ for some $s > 0$, then $\mathbb{E}(X^k) < \infty$ for all $k = 1, 2, 3, \ldots$.

Problem 3. Suppose the total claim amount X of an insurance company is a compound Poisson variable. The expected number of claims is 60. Let Z be the size of an individual claim. We assume that

$$\mathbb{P}(Z = 5) = \mathbb{P}(Z = 10) = \mathbb{P}(Z = 15) = \frac{1}{3}.$$

Suppose the company has a QS reinsurance contract for all claims. In the QS contract, the reinsurer pays portion $r = 1/2$ of every claim. The company also has an XL reinsurance contract for the remaining part after the QS reinsurance with XL limit 5.

Determine the risk premium of the insurer.

Problem 4. Consider the setting of the classical ruin theory where $\xi_1, \xi_2, \ldots$ is a sequence of independent and identically distributed real valued random variables and $Y_n = \xi_1 + \cdots + \xi_n$ is a random walk, $n = 1, 2, \ldots$. The random walk models the cumulative loss of an insurance company. Denote the initial capital of the company by U_0 and let $T = T(U_0)$ be the ruin time of the company. Let c be the cumulant generating function of ξ_1.

a) Give an example of a distribution of ξ_1 for which the Lundberg exponent does not exist.

b) Suppose the Lundberg exponent R exists in the setting of the problem. Assume further that $c(s) < \infty$ for all $s \in \mathbb{R}$ and that $\lim_{s \to \infty} c'(s) = \infty$.

   Show that

$$\lim_{U_0 \to \infty} \frac{\log \mathbb{P}\left(T \leq (U_0)^{2024}\right)}{U_0} = -R.$$