Path Integrals Quantization of Gauge Field Theories — Final exam
Spring 2014 — 5 March 2014

1. i) Write down (without derivation) the solution to the diffusion equation in terms of a path integral.

   ii)Present qualitative arguments that the trajectories of Brownian particles are nowhere differentiable, by showing that for a usual Brownian particle with the transition probability obeying the diffusion equation the notion of velocity, $\sqrt{\langle x^2 \rangle}/t$, is ill-defined when $t \to 0$.

2. List and justify the properties of operations over Grassmann variables (differentiation, integration, change of variables).

3. Write the expression for the transition amplitude in the Hamiltonian form for
$$H(p,x) = \frac{p^2}{2m} + V(x)$$
as a path integral in the phase space. Show how it can be transformed into the Feynman form as a path integral over coordinates x only.

4. Sketch the derivation and present the expression for the transition amplitude in quantum mechanics for a system with m (first class) constraints
$$\phi_a(q,p) = 0, \quad a = 1, ..., m.$$

5. What are the Faddeev-Popov ghosts and how do they appear in the path integral quantization of gauge field theory? Motivate their Grassmannian character. Is it possible to observe ghosts?

6. i) Consider the $SU(2)$ Yang–Mills theory in the *axial gauge*
$$n^\mu A^a_\mu(x) = 0, \quad a = 1, 2, 3,$$
where n_μ is a unit four-vector.

   Find $\Delta_{axial}(A^a_\mu(x))$. Are there any ghosts in the axial gauge?

   ii) Is it possible to have ghosts in the path integral quantization of electrodynamics?