MAST32017 Nonparametric Inference

Exam 16.12.2024

The only materials you are allowed to have at your desk during the exam are writing instruments and approved calculators.

1. Explain shortly but precisely the following concepts:

   a) Distribution-free test statistic

   b) Kendall's sample rank correlation coefficient

   c) Jonckheere–Terpstra statistic

2. A clinical trial was conducted to investigate the effect of clofibrate treatment on plasma fibrinogen levels in middle-aged men with high serum cholesterol level. The fibrinogen levels were measured in the beginning of study and after 8 weeks of treatment. The results are presented in the following table.

Patient	Fibrinogen level		Difference (d_i)	Y_i	$\lvert d_i \rvert$	$R(\lvert d_i \rvert)$
	Before (x_i)	After (y_i)				
1	379	325	-54	0	54	6
2	351	333	-18	0	18	3
3	370	323	-47	0	47	5
4	381	370	-11	0	11	2
5	349	354	5	1	5	1
6	284	249	-35	0	35	4
7	380	315	-65	0	65	7

Assume that $d_1, \ldots, d_n$ is a random sample from a continuous symmetrical distribution with an unknown median Δ. We would like to test the hypothesis $H_0: \ \Delta = 0$ using both one-sample sign test and one-sample Wilcoxon test.

   a) Calculate the observed value of the sign test statistic and the one-sample Wilcoxon test statistic.

   b) Derive the exact distribution of the sign test statistic under null hypothesis.

   c) How would you derive the exact distribution of the one-sample Wilcoxon test statistic under null hypothesis?

3. Page's test (in a randomized complete block design): Assumptions, hypothesis (null hypothesis and alternative hypothesis), test statistic and derivation of the exact distribution of the test statistic under H_0.

4. Consider the straight line model

$$Y_i = \alpha + \beta x_i + e_i, \quad i = 1, \ldots, n,$$

where the x's are known constants and α and β are unknown parameters. We assume that the random variables $e_1, \ldots, e_n$ are a random sample from a continuous population that has median 0. We also assume that $x_1 < x_2 < \cdots < x_n$.

We would like to test the null hypothesis that the slope β is some specified value β_0, i.e.

$$H_0 : \beta = \beta_0.$$

a) Give the Theil's test statistic C.

b) How would you derive the exact distribution of the Theil's test statistic under null hypothesis?

each eq. prob.