University of Helsinki Master's Programme in Mathematics and Statistics Mathematical logic, spring 2019 1st mid-term exam 5.3.2019

1. Show directly from the definition of provability that

$$\{(A \to (C \to B)), (\neg C \to \neg A)\} \vdash (A \to B).$$

2. Show that

$$\not\vdash ((\exists v_0 P v_0 \land \exists v_0 Q v_0) \rightarrow \exists v_0 (P v_0 \land Q v_0)).$$

- 3. Let  $L = \{R\}$  be a vocabulary with a binary relation symbol R. Consider the L-structure  $(\mathbb{Z}, <)$ , where < is the natural order on  $\mathbb{Z}$ . Is the relation  $S = \{(a, a+1) : a \in 2\mathbb{Z}\}$  definable, where  $2\mathbb{Z}$  is the set of even integers? Justify your answer.
- 4. (a) Show that if L is a vocabulary,  $\varphi$  and  $\psi$  are logically equivalent L-formulas (i.e.,  $\varphi \leftrightarrow \psi$  is valid), and t is an L-term such that both  $FVF(t, v_0, \varphi)$  and  $FVF(t, v_0, \psi)$  hold, then also  $\varphi(t/v_0)$  and  $\psi(t/v_0)$  are logically equivalent.
  - (b) Give an example showing that the claim is not true without the FVF-assumptions.