Mathematical logic
16.12.2022

Answer 3 of the following 4 questions.

1. Suppose that the vocabulary L is empty and M is an L-structure such that the universe of M is a singleton. Show that for all assignments s for M and L-formulas ϕ, if $\neg$ does not appear in ϕ, then $M \models_s \phi$.

2. Suppose that L is a vocabulary that contains a constant symbol c, Σ is an L-theory which consists of sentences in which the constant c does not appear and ϕ is an L-formula in which only v_0 appears freely. Show without using Lemma on constants that if $\Sigma \vdash \phi(c/v_0)$, then $\Sigma \vdash \forall v_0 \phi$. You may use Completeness theorem and Substitution lemma.

3. Suppose $R \subseteq \mathbb{N}^3$ is such that for all primitive recursive $P \subseteq \mathbb{N}$ there are $m, k \in \mathbb{N}$ for which for all $n \in \mathbb{N}$, $n \in P$ iff $(n, m, k) \in R$. Show that R is not primitive recursive.

4. Let $L = \{R\}$ and $M = (\{0, 1, 2\}, R^M)$, where $R^M = \{(0, 1), (1, 2), (2, 0)\}$. Find an L-sentence ϕ such that $M \models \phi$ and $\{\phi\}$ is a complete L-theory.