Mathematical logic

Exam

15.12.2023

Answer 3 of the following 4 questions.

1. Give the deduction (i.e. prove without using Completeness theorem):

$$\{\forall v_0 R(v_0, c)\} \vdash \forall v_1 (c = v_1 \rightarrow \forall v_0 R(v_0, v_1)).$$

2. Let $M = (\mathbb{N}, S, 0)$, where $S : \mathbb{N} \rightarrow \mathbb{N}$ is such that for all $x \in \mathbb{N}$, $S(x) = x+1$. Show that $Th(M)$ is not $\aleph_0$-categorical.

3. Let Trm be the set of all Gödel numbers of L_{exp}-terms. Show that Trm is primitive recursive.

4. We define P-functions as follows:

(a) Functions $Z, U, id : \mathbb{N} \rightarrow \mathbb{N}$ are P-functions, where $Z(x) = 0$, $U(x) = 1$ and $id(x) = x$ for all $x \in \mathbb{N}$,

(b) if f and g are P-functions, then also $f+g$ and fg are P-functions, where $(f+g)(x) = f(x) + g(x)$ and $(fg)(x) = f(x)g(x)$ for all $x \in \mathbb{N}$.

Show that $f : \mathbb{N} \rightarrow \mathbb{N}$ is a P-function iff there are $n \in \mathbb{N}$ and $a_0, ..., a_n \in \mathbb{N}$ such that for all $x \in \mathbb{N}$, $f(x) = \sum_{i=0}^{n} a_i x^i$.