Mathematical logic I

Exam

20.10.2026

Answer 4 of the following 5 questions.

1. Let S be the set of all (unary) functions $\mathbb{N} \to \mathbb{N}$, $f_0 \in S$ be such that $f_0(n) = n$ for all $n \in \mathbb{N}$ and $f_1 \in S$ such that $f_1(n) = 1$ for all $n \in \mathbb{N}$. Let $B = \{f_0, f_1\}$ and $F = \{f_+\}$, where $f_+ : S^2 \to S$ is such that $f_+(g_0, g_1) = g_2$ iff for all $n \in \mathbb{N}$, $g_2(n) = g_0(n) + g_1(n)$. Let $LF = cl_S(B, F)$. Show that for all $g \in LF$, there are $m, k \in \mathbb{N}$ such that for all $n \in \mathbb{N}$, $g(n) = mn + k$.

2. Is $(p_0 \to (p_1 \to \neg p_1 p_0))$ a propositional formula? Do not forget to prove your answer.

3. Show that $\{\forall v_0(\phi \to \psi), \exists v_0 \phi\} \models \exists v_0 \psi$.

4. Let $M = (\{0, 1, 2, 3\}, R^M)$, where $R^M = \{(1, 0), (1, 2), (1, 3), (2, 3), (3, 2)\}$. What are the definable elements of M? Again, prove your answer.

5. Suppose that v_0 does not occur free in ψ (i.e. $v_0 \notin FV(\psi)$). Give the deduction

$$\{\forall v_0(\phi \to \psi)\} \vdash \exists v_0 \phi \to \psi.$$