√ **1.** (i) Determine all complex roots of the equation $z^4 = -1$.

√ (ii) Write $z = x + iy$ in the usual way for $z \in \mathbf{C}$. Determine all holomorphic functions $f(z) = u(z) + iv(z) : \mathbf{C} \to \mathbf{C}$ (here u is the real part and v the imaginary part of f) such that

$$u(z) = x - y \qquad \text{for all} \quad z \in \mathbf{C}.$$

√ **2.** (i) Find a Möbius transformation $f : \hat{\mathbf{C}} \to \hat{\mathbf{C}}$ such that $f(0) = 1$, $f(1) = -1$ and $f(-1) = 0$.

(ii) Show that there exists no conformal bijection $f : \mathbf{C} \to \mathbf{D} = \{z \,:\, |z| < 1\}$.

3. Compute the contour integrals

√ (i) $\displaystyle\int_{\partial B(0,2)} \frac{e^z}{z} dz.$

(ii) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin(z)}{1+z^2} dz.$

4. (i). Explain (state explictly a theorem) what is maximum principle for holomorphic functions. No proofs are needed. √

√ (ii) Assume that function $f : \mathbf{C} \to \mathbf{C}$ is holomorphic and satisfies $|f(z)| \leq |e^z|$ for every $z = x + iy \in \partial B(0,1)$. Show that then $|f(0)| \leq 1$.