Axiomatic set theory
Exam
3.5.2024

Answer 3 of the following 4 questions.
You can use all the results including the exercises from the course except, obviously, the result/exercise asked in the question in the cases that this applies.

1. Suppose that κ is an uncountable limit cardinal and for all $\lambda < \kappa$, $2^\lambda \leq \kappa$. Show that $\kappa^{cf(\kappa)} = 2^\kappa$.

2. Show that for all formulas $\phi(x)$, $x = (x_1, ..., x_n)$, and $\alpha \in On$, there is $\beta \in On$ such that $\beta \geq \alpha$ and for all $a \in (L_\beta)^n$, $\phi^L(a)$ holds iff $\phi^{L_\beta}(a)$ holds. Conclude that Separation axiom is true in L.

3. Prove Δ-lemma.

4. Show that $Con(ZFC)$ implies $Con(ZFC + 2^\omega = \omega_2 + 2^{\omega_1} \geq \omega_4)$