Topology II
Exam December 18, 2023
Exam time 14.00-17.00

Problems

p1. Let $\mathbb{R}/\mathbb{Q}$ be the quotient space with quotient topology, where $\mathbb{R}$ has the standard Euclidean topology and the quotient is given by equivalence relation $\sim$ satisfying $x \sim y$ if $x = y$ or $x, y \in \mathbb{Q}$. Is the quotient space $\mathbb{R}/\mathbb{Q}$ separable?

p2. Let X and Y be topological spaces, $a \in X$, $b \in Y$. Show that

$$C(a, X) \times C(b, Y) \subset C((a, b), X \times Y),$$

where $C(z, Z)$ is the z-component of Z for $z \in Z$. Here $X \times Y$ has the product topology.

p3. Let X be a locally compact space and let $A \subset X$ be a closed subset. Show that A is a locally compact space with respect to the relative topology.

p4. Let $X = \{(x, y) \in \mathbb{R}^2 : |x| = |y| \le 1\}$ be a topological space with relative topology induced from $\mathbb{R}^2$. Is X an absolute retract? Justify your answer.